CIMT Further Statistics p48 Ex 3C

1. 60% collect prescriptions.

out of 12, 10 collected prescription

let X = no. people who collect prescription

$X \sim B(12, p)$

H_0: no. prescription collectors is 60%. $(p = 0.6)$

H_1: no. prescription collectors is $>$ 60% $(p > 0.6)$

Assume H_0 to be true.

$\alpha = 5\%$

one tailed test

$X \sim B(12, 0.6)$

$P(X \geq 10) = 0.083443$

> 0.05

So we do not have evidence to reject H_0 and conclude the proportion of prescription collectors is 60%.

OR

let Y be normal approximation to X

$Y \sim N(7.2, 2.88)$

this is questionable as $np > 5$ but $nq \not> 5$

let $\frac{Y}{12}$ = proportion who collect prescription.

$\frac{Y}{12} \sim N\left(\frac{7.2}{12}, \frac{2.88}{12^2}\right)$

we observed a proportion of $\frac{10}{12}$

$P\left(\frac{Y}{12} > \frac{10}{12}\right) = P\left(Z > \frac{\frac{10}{12} - \frac{7.2}{12}}{\sqrt{\frac{2.88}{12^2}}}\right)$

$= P(Z > 1.64992..)$

$= 0.04948$

< 0.05

Here we conclude that we have evidence against H_0 and the pharmacist's claim might well be correct.

However, this is questionable as $nq \not> 5$ when we approximated $B(12, 0.6)$ with $N(7.2, 2.88)$

2.

14 out of 30 bought comic regularly.

Is true proportion 0.35 ?

let: X = no. people who buy comic regularly

$X \sim B(30, p)$

$H_0 : p = 0.35$

$H_1 : p \neq 0.35$

Assume H_0 to be true

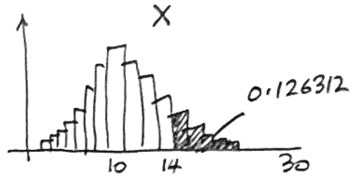
$\alpha = 10\%$

two tailed test

METHOD 1

$X \sim B(30, 0.35)$

$P(X \geq 14) = 0.126312\ldots$ from binomCdf (30, 0.35, 14, 30)



p-value $= 2 \times P(X \geq 14)$

$= 0.252624$

> 0.10

Hence, we don't have evidence to reject H_0

We do not have evidence that the proportion of people who buy a comic regularly is not 35%

<u>OR</u>

METHOD 2.

if $X \sim B(30, 0.35)$

approximate with $Y \sim N(30 \times 0.35, 30 \times 0.35 \times 0.65)$ as. $30 \times 0.35 > 5$ and $30 \times 0.65 > 5$ so should be good.

so $\frac{Y}{30} \sim N\left(0.35, \frac{30 \times 0.35 \times 0.65}{30^2}\right)$ where $\frac{Y}{30}$ = proportion of people who buy comic regularly.

$\frac{Y}{30} \sim N\left(0.35, \frac{0.35 \times 0.65}{30}\right)$

so $P\left(\frac{Y}{30} > \frac{14}{30}\right) = P\left(Z > \frac{\frac{14}{30} - 0.35}{\sqrt{\frac{0.35 \times 0.65}{30}}}\right)$

$= P(Z > 1.33973)$

$= 0.090167$

so p-value $= 2 \times 0.090167$

$= 0.180334$

> 0.10

Hence we don't have evidence to reject H_0

We do not have evidence that the proportion of people who buy a comic regularly is not 35%

3.

out of 30 people, Like = 21

Indif = 5

Dislike = 4

claim : more than half like it.

let X = no. people who like it

$X \sim B(30, p)$

$H_0 : p = 0.5$

$H_1 : p > 0.5$

Assume H_0 to be true $\qquad \alpha = 5\%$, one tail test

METHOD 1

$X \sim B(30, 0.5)$

$P(X \geq 21) = 0.021387$ from binomCdf$(30, \frac{1}{2}, 21, 30)$

< 0.05

So we have evidence to reject H_0 and conclude that the claim is plausible that over 50% like the new brand of coffee.

OR

METHOD 2.

if $X \sim B(30, 0.5)$

Approximate by $Y \sim N(30 \times 0.5, 30 \times 0.5 \times 0.5)$ as $30 \times 0.5 = 15 > 5$ ✓ check $np > 5$, $nq > 5$

So $\frac{Y}{30} \sim N\left(0.5, \frac{0.5 \times 0.5}{30}\right)$ where $\frac{Y}{30}$ = proportion of people who like it.

So p-value $= P\left(\frac{Y}{30} > \frac{21}{30}\right)$

$= P\left(Z > \dfrac{\frac{21}{30} - 0.5}{\sqrt{\frac{0.5 \times 0.5}{30}}}\right)$

$= P(Z > 2.19089)$

$= 0.01423$

< 0.05

So we have evidence to reject H_0 and conclude that the claim is plausible that over 50% like the new brand of coffee.

4.

out of 25×24 tins, we had 8 damaged

claim that less than 2% damaged.

let X = no. damaged tins

$X \sim B(600, p)$

$H_0: p = 0.02$

$H_1: p < 0.02$

Assume H_0 to be true

$\alpha = 5\%$. one tail test

METHOD 1

$X \sim B(600, 0.02)$

$P(X \leq 8) = 0.152388$ from binomcdf $(600, 0.02, 0, 8)$

p-value $= 0.152388$

> 0.05

So we do not have evidence to reject H_0

We don't have evidence that less than 2% are damaged.

OR

METHOD 2

if $X \sim B(600, 0.02)$

approximate with $Y \sim N(600 \times 0.02, 600 \times 0.02 \times 0.98)$ as $600 \times 0.02 = 12 > 5$, $600 \times 0.98 > 5$ ✓

then $\frac{Y}{600} \sim N\left(0.02, \frac{0.02 \times 0.98}{600}\right)$ where $\frac{Y}{600}$ = proportion of damaged tins

so p-value $= P\left(\frac{Y}{600} < \frac{8}{600}\right)$

$$= P\left(Z < \frac{\frac{8}{600} - 0.02}{\sqrt{\frac{0.02 \times 0.98}{600}}}\right)$$

$$= P(Z < -1.16642)$$

$$= 0.121722$$

$$> 0.05$$

So we don't have evidence to reject H_0,

We do not have evidence to suggest that less than 2% are damaged.

5. 223 out of 500 have double glazing

claim that 40% have double glazing

let X = no. of houses with double glazing

$X \sim B(500, p)$

$H_0 : p = 0.4$

$H_1 : p \neq 0.4$

Assume H_0 to be true

$\alpha = 5\%$ two tail test

METHOD 1

so $X \sim B(500, 0.4)$

$P(X \geq 223) = 0.020408$ from binomCdf(500, 0.4, 223, 500)

$$\begin{aligned} p\text{-value} &= 2 \times 0.020408 \\ &= 0.040815 \\ &< 0.05 \end{aligned}$$

So we have evidence to reject H_0 and that it's not true that 40% of houses have double glazing.

OR

METHOD 2

if $X \sim B(500, 0.4)$

approximate with $Y \sim N(500 \times 0.4, 500 \times 0.4 \times 0.6)$ as $500 \times 0.4 = 200 > 5$ and $500 \times 0.6 = 300 > 5$ ✓

so $\frac{Y}{500} \sim N\left(0.4, \frac{0.4 \times 0.6}{500}\right)$ where $\frac{Y}{500}$ = proportion that have double glazing.

$$\begin{aligned} p\text{-value} &= 2 \times P\left(\frac{Y}{500} > \frac{223}{500}\right) \\ &= 2 \times P\left(Z > \frac{\frac{223}{500} - 0.4}{\sqrt{\frac{0.4 \times 0.6}{500}}}\right) \\ &= 2 \times P(Z > 2.0996) \\ &= 2 \times 0.017882 \\ &= 0.035764 \\ &< 0.05 \end{aligned}$$

so we have evidence to reject H_0 and that it's not true that 40% of houses have double glazing.

6. 72 out of 135 shots in basket

claim that scoring ability is 0·45

let X = no. shots in basket

$X \sim B(135, p)$

$H_0: p = 0{\cdot}45$

$H_1: p > 0{\cdot}45$

Assume H_0 to be true

$\alpha = 5\%$ one-tailed test

METHOD 1

$X \sim B(135, 0{\cdot}45)$

$P(X \geq 72) = 0{\cdot}031761$ from binomCdf(135, 0·45, 72, 135)

$< 0{\cdot}05$

so we have evidence to reject H_0 and conclude that the course has improved her shooting ability, as her proportion of shots in the basket is more than 0·45.

OR

METHOD 2

if $X \sim B(135, 0{\cdot}45)$

then approx with $Y \sim N(135 \times 0{\cdot}45,\ 135 \times 0{\cdot}45 \times 0{\cdot}55)$ as $135 \times 0{\cdot}45 > 5$, $135 \times 0{\cdot}55 > 5$ ✓

so $\frac{Y}{135} \sim N\left(0{\cdot}45,\ \frac{0{\cdot}45 \times 0{\cdot}55}{135}\right)$ where $\frac{Y}{135}$ = proportion of shots in basket

so p-value $= P\left(\frac{Y}{135} > \frac{72}{135}\right)$

$= P\left(Z > \dfrac{\frac{72}{135} - 0{\cdot}45}{\sqrt{\frac{0{\cdot}45 \times 0{\cdot}55}{135}}}\right)$

$= P(Z > 1{\cdot}94625)$

$= 0{\cdot}025812$

$< 0{\cdot}05$

so we have evidence to reject H_0 and conclude that the course has improved her shooting ability, as the proportion of shots in the basket is more than 0·45