CIMT Further Stats p75 Ex 4D

1. X = difference in scores. (difference = theory − practical)

assume X to be normally distributed (as given)

$X \sim N(\mu, \sigma^2)$

Student	A	B	C	D	E	F	G	H	I	J	K.
Theory	30	42	49	50	63	38	43	36	54	42	26
Practical	52	58	42	67	94	68	22	34	55	48	17
T−P	−22	−16	7	−17	−31	−30	21	2	−1	−6	9

$H_0 : \mu = 0$

$H_1 : \mu \neq 0$

Assume H_0 to be true

$\alpha = 5\%$

two tail test

we estimate σ from (T−P)'s $S_{n-1} = 16.9131$ and we have $n = 11$ (small) so we perform a t-test on the differences. (so turning this into a single sample t-test)

we also know that $\bar{x} = -7.63636$

so $X \sim N(0, \sigma^2)$

$\bar{X} \sim N(0, \frac{\sigma^2}{11})$ where $\bar{X}$ = mean difference

so $\dfrac{\bar{X} - 0}{\sqrt{\frac{S^2}{11}}} \sim t_{10}$ as we estimate σ with S_{n-1}

$$\text{p-value} = 2 \times P\left(t_{10} < \frac{-7.63636 - 0}{\sqrt{\frac{16.9131^2}{11}}}\right)$$

$$= 2 \times P(t_{10} < -1.49747)$$

$$= 2 \times 0.082576$$

$$= 0.165153$$

$$> 0.05$$

No evidence to reject H_0.

We do not have evidence to suggest that the mean difference in scores is not zero.

This suggests that the scores on the Theory and Practical papers are similar.

2.

X = differences in sales figures. (where difference = after − before)

Assume X to be normally distributed

let $X \sim N(\mu, \sigma^2)$

Region	A	B	C	D	E	F	G	H	I	J	K
Before	2·4	2·6	3·9	2·0	3·2	2·2	3·3	2·1	3·1	2·2	2·8
After	3·0	2·5	4·0	4·1	4·8	2·0	3·4	4·0	3·3	4·2	3·9
A−B	0·6	−0·1	0·1	2·1	1·6	−0·2	0·1	1·9	0·2	2	1·1

$H_0 : \mu = 0$

$H_1 : \mu > 0$

Assume H_0 to be true

$\alpha = 5\%$

one tail test

by looking at "After-Before" we create a single sample of size $n = 11$

we estimate σ from $s_{n-1} = 0.907043$, and we have small n, so we perform a single sample t-test on the differences.

we also know that $\bar{x} = 0.854545$.

so $X \sim N(0, \sigma^2)$

$\bar{X} \sim N\left(0, \frac{\sigma^2}{11}\right)$ where $\bar{X}$ = mean difference

$$\frac{\bar{X} - 0}{\sqrt{\frac{S^2}{11}}} \sim t_{10}$$ as we estimate σ with s_{n-1}

$$p\text{-value} = P\left(t_{10} > \frac{0.854545 - 0}{\sqrt{\frac{0.907043^2}{11}}}\right)$$

$$= P(t_{10} > 3.12467)$$

$$= 0.005394$$

$$< 0.05$$

Hence we have evidence to reject H_0 and so the mean difference in sales is greater than zero, which tells us that sales have increased after the campaign.

3. $X=$ difference in accuracy. (where difference = crackshot − fastfire)

Assume X is normally distributed

Competitors	A	B	C	D	E	F	G	H	I	J
Crackshot	93	99	90	86	85	94	87	91	96	79
Fastfire	87	91	86	87	78	95	89	84	88	74
C−F	6	8	4	−1	7	−1	−2	7	8	5

$X \sim N(\mu, \sigma^2)$

$H_0: \mu = 0$

$H_1: \mu > 0$

Assume H_0 to be true

$\alpha = 5\%$

one tail test

we calculate "C−F" to give a single sample of size 10

we estimate σ to be $s_{n-1} = 3.95671$, and thus we do a single sample t-test

we also know $\bar{x} = 4.1$

so $X \sim N(0, \sigma^2)$

$\bar{X} \sim N(0, \frac{\sigma^2}{10})$ where $\bar{X}$ = mean difference in accuracy.

$\frac{\bar{x} - 0}{\sqrt{\frac{s^2}{10}}} \sim t_9$ as we estimate σ with s_{n-1}

$$p\text{-value} = P\left(t_9 > \frac{4.1 - 0}{\sqrt{\frac{3.95671^2}{10}}}\right)$$

$= P(t_9 > 3.2768)$

$= 0.004789$

< 0.05

Hence we have evidence to reject H_0 and conclude that the mean difference in accuracy is greater than zero. This suggests that Crackshot shotguns are more accurate.

4. X = difference in scores (where difference = third − fourth)

Assume X to be normally distributed

so $X \sim N(\mu, \sigma^2)$

Competitor	A	B	C	D	E
Third	76	75	72	75	79
Fourth	70	73	71	68	76
T-F	6	2	1	7	3

$H_0: \mu = 0$ (no diffence in scores)

$H_1: \mu \neq 0$ (one round is better than the other)

Assume H_0 is true

$\alpha = 5\%$

two tail test

we calculated '3rd round − 4th round' to give a single sample of size $n=5$

we estimate σ with $s_{n-1} = 2.58844$, and we have small n, so perform a single sample t-test

we also know $\bar{x} = 3.8$

so $X \sim N(0, \sigma^2)$

$\bar{X} \sim N(0, \frac{\sigma^2}{5})$

$\frac{\bar{X} - 0}{\sqrt{\frac{s^2}{5}}} \sim t_4$ as we estimate σ with s_{n-1}

$$p\text{-value} = 2 \times P\left(t_4 > \frac{3.8 - 0}{\sqrt{\frac{2.58844^2}{5}}}\right)$$

$= 2 \times P(t_4 > 3.2827)$

$= 2 \times 0.015212$

$= 0.030423$

< 0.05

Hence, we have sufficient evidence to reject H_0 and conclude that the mean difference in scores is non-zero, telling us that the 3rd and 4th round scores are different.

(we conjecture that they are better/lower scoring on their 4th round)

5. X = difference in number of words recalled (where difference = "1 hr" − "24 hr")

assume X to be normally distributed

$X \sim N(\mu, \sigma^2)$

Student	A	B	C	D	E	F	G	H	I	J	K	L
1hr	14	9	18	12	13	17	16	16	19	8	15	7
24hr	10	6	14	6	8	10	12	10	14	5	10	5
"1−24"	4	3	4	6	5	7	4	6	5	3	5	2

$H_0 : \mu = 5$

$H_1 : \mu \neq 5$

We assume H_0 to be true

$\alpha = 5\%$

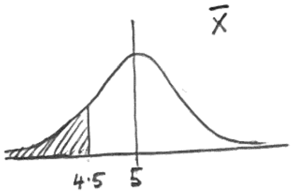
two tailed test

we calculate "1hr − 24hr" to give single sample of size $n=12$

we estimate σ from $s_{n-1} = 1.446$, and as n is small, we perform single sample t-test

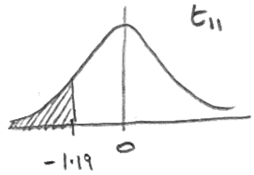
we also know $\bar{x} = 4.5$

so $X \sim N(5, \sigma^2)$



$\bar{X} \sim N(5, \frac{\sigma^2}{12})$ where $\bar{X}$ = mean difference

so $\dfrac{\bar{X}-5}{\sqrt{\frac{s^2}{12}}} \sim t_{11}$ as we estimate σ with s_{n-1}



so p-value $= 2 \times P\left(t_{11} < \dfrac{4.5-5}{\sqrt{\frac{1.446^2}{12}}}\right)$

$= 2 \times P(t_{11} < -1.19782)$

$= 2 \times 0.128079$

$= 0.256159$

> 0.05

Hence we donot have evidence to reject H_0

We do not have evidence to suggest that the mean difference in words is not 5.

And so the study seems to show that the number of words recalled after 1 hour exceeds that recalled after twenty four hours by a mean of 5 words

6. X = difference in temperature (where difference = satelite − ground)

as each temperature (Ground and Satellite) are normally distributed, we can assume that their differences are also normally distributed.

Site	1	2	3	4	5	6	7	8	9	10	11
Ground	4·6	17·3	12·2	3·6	6·2	14·8	11·4	14·9	9·3	10·4	7·2
Satellite	4·7	19·5	12·5	4·2	6·0	15·4	14·9	17·8	9·7	10·5	7·4
S−G	0·1	2·2	0·3	0·6	−0·2	0·6	3·5	2·9	0·4	0·1	0·2

so $X \sim N(\mu, \sigma^2)$

$H_0: \mu = 0$

$H_1: \mu > 0$

We assume H_0 is true,

$\alpha = 5\%$

one tail test

we calculate 'Satellite − Ground' to give a single sample of $n = 11$, for which we estimate σ to be $s_{n-1} = 1{\cdot}27129$, and so we perform a t-test on this single sample

we also know $\bar{x} = 0{\cdot}972727$

so $X \sim N(0, \sigma^2)$

$\bar{X} = N\left(0, \frac{\sigma^2}{11}\right)$ where $\bar{x}$ = mean temperature difference

so $\dfrac{\bar{X} - 0}{\sqrt{\frac{s^2}{11}}} \sim t_{10}$ as we estimate σ with s_{n-1}

$$\text{p-value} = P\left(t_{10} > \frac{0{\cdot}972727 - 0}{\sqrt{\frac{1{\cdot}27129^2}{11}}}\right)$$

$$= P(t_{10} > 2{\cdot}53771)$$

$$= 0{\cdot}014739$$

$$< 0{\cdot}05$$

Hence we have evidence to reject H_0 and conclude the mean difference in temperature is greater than zero, which means that Satellite sensors give higher readings than ground sensors.