CIMT Further Statistics p54 Ex 3.6

3.

x	0	1	2	3	4	5
f	4	13	27	22	13	1
e	2.5	12.5	25	25	12.5	2.5

where X = number of correct forecasts, out of 5.

$\sum f = 80$

H_0: data fits $B(5, 0.5)$ model

H_1: data does not fit $B(5, 0.5)$

Assume H_0 to be true, $\alpha = 5\%$, one tail test

Note we have $\frac{1}{3}$ of $f_e < 5$, so we must combine categories...

x	0–1	2	3	4	5
f	17	27	22	13	1
e	15	25	25	12.5	2.5

$\nu = 5 - 1 = 4$, $X^2 = \sum \frac{(O-E)^2}{E} = 1.70667$

$P(X^2 > 1.70667) = 0.789506$
> 0.05

OR

x	0	1	2	3	4–5
f	4	13	27	22	14
e	2.5	12.5	25	25	15

$\nu = 5 - 1 = 4$, $X^2 = \sum \frac{(O-E)^2}{E} = 1.50667$

$P(X^2 > 1.50667) = 0.82546$
> 0.05

So do not reject H_0

We do not have evidence to suggest that Miss Fortune is not correctly forecasting with a probability of 0.5.

with $X \sim B(5, 0.5)$ and $P(X=5) = (0.5)^5 = 0.03125$

let Y = net winnings from £1 bet (assuming that for a win, the initial fee is not refunded)

y	-1	99
$P(Y=y)$	0.96875	0.03125

$E(Y) = -0.96875 + 99 \times 0.03125$

$= 2.125$

Hence, in the long run, we expect to have net winnings of about £2.13

As this is positive, I would advise Miss Fortune to continue with this competition.

4. we are being asked to test if the population mean is greater than 3.

this suggests we need the sample mean, $\bar{x} = \frac{1}{10}(3+6+5+\dots+1+4)$
$= 4$

and we need the distribution of the sample mean....

if X = no. of visits to reactor room in 1 year, then $X \sim Po(\lambda)$.

and $H_0: \lambda = 3$
$H_1: \lambda > 3$

so if we assume H_0 to be true, then $X \sim Po(3)$

now the sample mean, $\bar{X} = \frac{1}{10}(X_1 + X_2 + \dots + X_{10})$ where $X_i \sim Po(3)$

we know that $X_1 + X_2 + \dots + X_{10} \sim Po(3+3+\dots+3)$
$\Rightarrow X_1 + \dots + X_{10} \sim Po(30)$

However, we do not know how to work out the distribution of $\bar{X}$ if $X_1 + \dots + X_{10} \sim Po(30)$

But we can approximate it with a Normal Distribution, $Y \approx N(30, 30)$ as $30 > 10$.

therefore $\bar{X} = \frac{1}{10}Y$, approximately, and we consider $\bar{X}$ to be continuous, rather than discrete.

so $\bar{X} \approx N\left(\frac{30}{10}, \frac{30}{10^2}\right)$

$\Rightarrow \bar{X} \approx N\left(3, \frac{30}{10^2}\right)$

so p-value $= P(\bar{X} \geq 4)$

$= P\left(Z \geq \dfrac{4-3}{\sqrt{\frac{30}{10^2}}}\right)$

$= P(Z \geq 1.82574)$

$= 0.033945$ by normcdf(1.82574, 9E99)

< 0.05

So, at a 5% level of significance, we have evidence to reject H_0 and conclude that the annual mean visits per year to the reactor room is greater than 3.

Note: The problems arose in the above solution when we tried to "divide" the $Po(30)$ distribution by 10, to work out the distribution of $\bar{X}$.
We can avoid this by not looking at the sample mean, but rather by looking at the sample total.
See overleaf for an alternative solution using this method.

4 (alternative solution)

work out sample total $= 3+6+\dots+1+4 = 40$.

if X = no. visits per year per person, then $X \sim Po(\lambda)$

and $H_0 : \lambda = 3$
$H_1 : \lambda > 3$

so if we assume H_0 to be true, then $X \sim Po(3)$

let T = total visits from 10 independent people

$\Rightarrow T = X_1 + \dots + X_{10}$ where $X_i \sim Po(3)$

$\Rightarrow T \sim Po(10 \times 3)$

$T \sim Po(30)$

so p-value $= P(T \geq 40)$

$= 1 - P(T \leq 39)$

$= 1 - 0.95374$ from poissCdf (30, 0, 39)

$= 0.046253$

< 0.05

so, at a 5% level of significance, we have evidence to reject H_0 and conclude that the annual mean visits per year to the reactor room is greater than 3.

Note: the p-value of this alternative solution was different to the first solution, as there was no normal approximation involved.

6. the question appears to be about mean numbers of misprints

this suggests we need the sample mean, $\bar{x} = \frac{1}{16}(0+3+\dots+3+1)$

$$\bar{x} = \frac{45}{16}$$

and we would need the distribution of this sample mean....

if X = no. of misprints per page, so $X \sim Po(\lambda)$

if $H_0 : \lambda = 4$
$H_1 : \lambda < 4$

if we assume H_0 to be true, then $X \sim Po(4)$

now the sample mean, $\bar{X} = \frac{1}{16}(X_1 + \dots + X_{16})$ where $X_i \sim Po(4)$

we know that $X_1 + \dots + X_{16} \sim Po(4 \times 16)$

$X_1 + \dots + X_{16} \sim Po(64)$

However, we do not know how to work out the distribution of $\bar{X}$, as it will involve dividing a Poisson distribution by 16

But we can approximate $X_1 + \dots + X_{16}$ with a normal distribution, $Y \sim N(64, 64)$ as $64 > 10$

Therefore $\bar{X} = \frac{1}{16} Y$, approximately, and we now consider $\bar{X}$ to be continuous

so $\bar{X} \approx N\left(\frac{64}{16}, \frac{64}{16^2}\right)$

$\bar{X} \approx N\left(4, \frac{64}{16^2}\right)$

so p-value $= P\left(\bar{X} \leq \frac{45}{16}\right)$

$$= P\left(Z < \frac{\frac{45}{16} - 4}{\sqrt{\frac{64}{16^2}}}\right)$$

$= P(Z < -2.375)$

$= 0.008774$ by normCdf$(-9E99, -2.375)$

< 0.01

So at the 1% level of significance, we have evidence to reject H_0 and conclude that there are fewer than a mean of 4 misprints per page.

Note: the problems arose here when we had to determine the distribution of $Po(64)$ "divided by 16".

We can avoid this by not looking at the sample mean, but rather by looking at the sample total. See overleaf for an alternative solution.

6 (alternative solution)

work out sample total $= 0 + 3 + \dots + 3 + 1$

$= 45$

if X = no. misprints per page, then $X \sim Po(\lambda)$

$H_0 : \lambda = 4$

$H_1 : \lambda < 4$

if we assume H_0 to be true, then $X \sim Po(4)$

let T = total misprints over 16 pages

$T \sim Po(16 \times 4)$

$T \sim Po(64)$

p-value $= P(T \leqslant 45)$

$= 0.007805$ by poissCdf(64, 0, 45)

< 0.01

So we have evidence at the 1% level to reject H_0 and conclude that there are fewer than a mean of 4 misprints per page.

Note: the differing p-values between the two solutions arises as one involved a normal approximation and the other did not.

(3rd alternative approach)

let X = no. misprints per page

Consider a χ^2 goodness of fit test, ...

x	0	1	2	3	4	5
f_o	1	2	3	5	3	2

$\Sigma f_o = 16$

we can anticipate that many of the expected frequencies - if we tried to fit a Poission distribution - would be small (<1 or <5)

Hence, as $\Sigma f_o = 16$ is not sufficiently large, we would not be able to reliably progress with a χ^2 goodness of fit test.

10. X = no. chocolates that have hard centres.

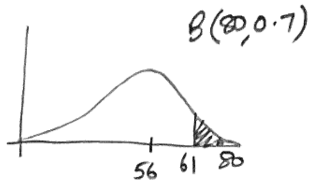
$X \sim B(80, p)$

$H_0: p = 0.7$
$H_1: p \neq 0.7$

Assume H_0 to be true.

$\alpha = 10\%$

two tail test



so $X \sim B(80, 0.7)$

p-value $= 2 \times P(X \geq 61)$
$= 2 \times 0.135223$ from binomCdf(80, 0.7, 61, 80)
$= 0.270447$
> 0.10

OR as $np > 5$ and $nq > 5$, we can do Normal Approximation, Y to X
$Y \sim N(56, 16.8)$
$P(X \geq 61) \simeq P(Y > 60.5)$
$= P\left(Z > \frac{60.5 - 56}{\sqrt{16.8}}\right) \approx 0.136127$

We do not have evidence to reject H_0, and we conclude that there are 70% with hard centres

---

let X = diameter of one chocolate
we are to assume that X is normally distributed
$X \sim N(\mu, \sigma^2)$

$H_0: \mu = 275$
$H_1: \mu \neq 275$

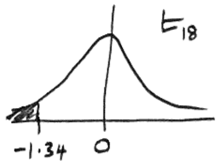
Assume H_0 is true
$\alpha = 10\%$
two tail test

as $n = 19$, and we estimated σ with $s_{n-1} = 6.47216$, we will use t-test
we have $\bar{x} = 273$

so $X \sim N(275, \sigma^2)$

$\bar{X} \sim N\left(275, \frac{\sigma^2}{19}\right)$ where $\bar{X}$ = mean diameter of 19 chocolates, giving $\frac{\bar{X} - 275}{\sqrt{\frac{\sigma^2}{19}}} \sim N(0, 1^2)$

$t = \frac{\bar{x} - 275}{\sqrt{\frac{s^2}{19}}} \sim t_{18}$



$P\left(t_{18} < \frac{273 - 275}{\sqrt{\frac{6.47216^2}{19}}}\right) = P(t_{18} < -1.34697)$
≈ 0.097353

so p-value $= 2 \times 0.097353$
$= 0.194705$
> 0.10 so do not reject H_0.

We do not have evidence to suggest that the mean diameter is not 275 mm.

---

if we knew that $\sigma = 5$, we'd use a z-test as $\bar{X} \sim N\left(275, \frac{5^2}{19}\right)$ giving

p-value $= 2 \times P(\bar{X} \leq 273)$
$= 2 \times P\left(Z < \frac{273 - 275}{\sqrt{\frac{5^2}{19}}}\right)$
$= 2 \times P(Z < -1.74356) \approx 0.081236 < 0.10$, giving the opposite conclusion!

12. Modelled by Binomial as there are a fixed number of trials (5) and a suspected fixed proportion of the population who drive around, alone

So $n=5$ and $np = \dfrac{0\times0+1\times3+2\times12+3\times27+4\times26+5\times12}{80} = \dfrac{272}{80} = 3.4$, so $p = \dfrac{3.4}{5} = 0.68$

optional process

So we conduct a χ^2 goodness of fit test to check if Binomial Distribution is indeed valid:

x	0	1	2	3	4	5
f	0	3	12	27	26	12
e	0.27	2.85	12.1	25.8	27.4	11.6

merge as need to be >1

$\Rightarrow$

x	0-1	2	3	4	5
f	3	12	27	26	12
e	3.12	12.1	25.8	27.4	11.6

H_0: data modelled by $B(5, 0.68)$

H_1: data not binomially distributed as $B(5, 0.68)$

Assume H_0 to be true

$\alpha = 5\%$

one tail test

as $\Sigma f = 80$

as $p = 0.68$ estimated from data

$df = 5-1-1 = 3$

$X^2 = \Sigma \frac{(f-e)^2}{e} = 0.145809$

p-value $= P(X^2 > 0.145809) = 0.985823 \gg 0.05$ so do not reject H_0.

we do not have evidence to suggest that the data is not $B(5, 0.68)$ distributed.

X = no. cars with driver only

$X \sim B(n, p)$

$H_0: p = 0.6$

$H_1: p > 0.6$

Assume H_0 to be true.

$\alpha = 5\%$.

one-tail test.

Here $n = 80 \times 5 = 400$, so $X \sim B(400, 0.6)$

we observed 272 cars with driver only

p-value $= P(X \geq 272)$

$= 0.000568$

< 0.05

Hence, we have evidence to reject H_0 and conclude that more than 60% of cars have only a driver.

OR

as $np = 240 > 5$ and $nq = 160 > 5$ we can approx X with normal r.v., Y

$Y \sim N(240, 96)$

$P(X \geq 272) \approx P(Y > 271.5)$ by cc.

$= P\left(Z > \frac{271.5 - 240}{\sqrt{96}}\right)$

$= P(Z > 3.21496)$

$= 0.000652$

< 0.05

13 significance level = the threshold that you'll accept the possibility of incorrectly rejecting H_0 when it is, in fact, true.

critical region = the range of values of the test statistic that would lead to rejecting H_0

test statistic = the value of the random variable that represents the findings of the sample that was taken from the parent population.

X = time taken to travel from depot to factory,

we assume X are normally distributed

$X \sim N(\mu, \sigma^2)$

3	5 9 9
4	0 3 7
5	1 8

4|3 = 43.

$H_0: \mu = 40$

$H_1: \mu \neq 40$

Assume H_0 to be true

$\alpha = 5\%$

two tail test

we estimate σ with $s_{n-1} = 7.57816$ and as $n=8$ is small, we use t-test

we also have $\bar{x} = 44$

so $X \sim N(40, \sigma^2)$

$\bar{X} \sim N\left(40, \frac{\sigma^2}{8}\right)$ where $\bar{X}$ = mean time from sample of 8, giving $\frac{\bar{X}-40}{\sqrt{\frac{\sigma^2}{8}}} \sim N(0, 1^2)$

$t = \frac{\bar{X}-40}{\sqrt{\frac{s^2}{8}}} \sim t_7$

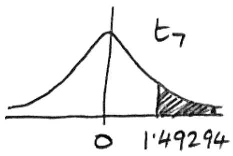
$p\text{-value} = 2 \times P\left(t_7 > \frac{44-40}{\sqrt{\frac{7.57816^2}{8}}}\right)$

$= 2 \times P(t_7 > 1.49294)$

$\approx 2 \times 0.089545$

≈ 0.179091

> 0.05 so we do not reject H_0



We do not have evidence to suggest that the mean time taken is not 40 mins.

The manager was right to remove the outlier of 180 as it was not a representative value of the quantity being investigated (ie. travel time) as it had been influenced by an unusual event.

If this value had been included, then the assumption that X was normally distributed would have come into question, as would the notion of all values in the sample being independent and identically distributed.

If all nine values had been considered, I would have progressed with a non-parametric test of the median travel time, such as a Wilcoxon Single Sample Rank Sum test, as this would not require assumptions of normality and the median is not adversely affected by outliers, in the same way as the mean is.

15. a) before, mean = 100s.

X = operating time

Assume X is normally distributed

$X \sim N(\mu, \sigma^2)$

$H_0: \mu = 100$

$H_1: \mu < 100$

Assume H_0 is true

$\alpha = 5\%$

one-tail test

we will estimate σ with $s_{n-1} = 3.49603$, as as $n = 10$ is small, we shall do a t-test

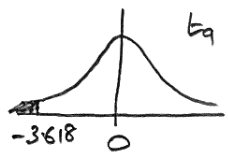
we also have $\bar{x} = 96$

so $X \sim N(100, \sigma^2)$

$\bar{X} \sim N(100, \frac{\sigma^2}{10})$ where $\bar{X}$ = mean operating time of sample of 10, giving $\frac{\bar{X}-100}{\sqrt{\frac{\sigma^2}{10}}} \sim N(0, 1^2)$

$t = \frac{\bar{X}-100}{\sqrt{\frac{s^2}{10}}} \sim t_9$

so p-value $= P\left(t_9 < \frac{96-100}{\sqrt{\frac{3.49603^2}{10}}}\right)$



$= P(t_9 < -3.61814)$

$= 0.002794$

< 0.05

so we have evidence to reject H_0 and conclude that the overhaul has improved the machine's operation as the mean operating time is less than 100s.

---

b) X = no. people purchasing magazine

$X \sim B(10000, p)$

$H_0: p = 0.35$

$H_1: p > 0.35$

Assume H_0 to be true. $\alpha = 1\%$. One tail test

we observe sample proportion to be $\frac{3615}{10000} = 0.3615$

$X \sim B(10000, 0.35)$

Approx X with $Y \sim N(10000 \times 0.35, 10000 \times 0.35 \times 0.65)$

$\frac{Y}{10000} \sim N\left(0.35, \frac{0.35 \times 0.65}{10000}\right)$ where $\frac{Y}{10000}$ = proportion of people buying mag.

so p-value $= P\left(\frac{Y}{10000} > 0.3615\right)$

$= P\left(Z > \frac{0.3615 - 0.35}{\sqrt{\frac{0.35 \times 0.65}{10000}}}\right)$

$= P(Z > 2.41106)$

$= 0.007953$

< 0.01

Hence we have evidence to reject H_0 and conclude that more than 35% of families purchased the magazine

18 X = weight of packet

$X \sim N(\mu, \sigma^2)$

$H_0 : \mu = 200$

$H_1 : \mu \neq 200$

Assume H_0 to be true

$\alpha = 5\%$

two tailed test

we shall estimate σ with $s_{n-1} = 11.7589$, as $n = 30$ we shall use a t-test

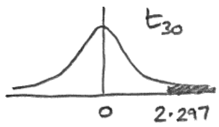
we also know $\bar{x} = 204.933$

so $X \sim N(200, \sigma^2)$

$\bar{X} \sim N(200, \frac{\sigma^2}{30})$ where $\bar{X}$ = mean weight of sample of 30, giving $\dfrac{\bar{X} - 200}{\sqrt{\frac{\sigma^2}{30}}} \sim N(0, 1^2)$

so $\dfrac{\bar{X} - 200}{\sqrt{\frac{s^2}{30}}} \sim t_{29}$ as we estimate σ with s_{n-1}

$p\text{-value} = 2 \times P\left(t_{29} > \dfrac{204.933 - 200}{\sqrt{\frac{11.7589^2}{30}}}\right)$



$= 2 \times P(t_{29} > 2.29792)$

$= 2 \times 0.014485$

$= 0.028971$

< 0.05

So we have evidence to reject H_0 and conclude that the mean weight is not 200g.

(we conjecture that it's now more than 200g)

X = no. packets less than 190g

$X \sim B(30, p)$

$H_0 : p = 0.15$

$H_1 : p \neq 0.15$

Assume H_0 to be true

$\alpha = 5\%$

two-tail test

we observe that 6 are less than 190g, $X \sim B(30, 0.15)$

$p\text{-value} = 2 \times P(X \geq 6)$

$= 2 \times 0.289424$

$= 0.578849$

> 0.05

So we don't have evidence to reject H_0,

we do not have evidence to suggest that the percentage of packets that weigh less than 190g is not 15%

19. a) X = no. accidents per 7 days

$X \sim Po(\lambda)$

$H_0: \lambda = 7$

$H_1: \lambda > 7$

Assume H_0 to be true

$\alpha = 5\%$

one tailed test

$p\text{-value} = P(X \geq 13)$

$= 1 - P(X \leq 12)$

$= 1 - 0.973$

$= 0.027$

< 0.05

so we have evidence to reject H_0 and conclude that the police are right that there are more than one accident per day.

b) X = no. in favour of proposals, who are house owners

no. of house owners $= 200 \times 0.75$

$= 150$

no. who in favour $= 70$

of the 70, no. who were house owners $= 58$

so, proportional of house owners in favour $= \frac{58}{150} \approx 0.38667$

so $X \sim B(150, p)$

$H_0: p = 0.40$

$H_1: p \neq 0.40$

Assume H_0 is true

$\alpha = 5\%$

two tailed test

$X \sim B(150, 0.40)$

let Y be normal approx to X, $Y \sim N(150 \times 0.4, 150 \times 0.4 \times 0.6)$

so $\frac{Y}{150} \sim N\left(0.4, \frac{0.4 \times 0.6}{150}\right)$ where $\frac{Y}{150}$ = proportion who own houses, who are in favour

$p\text{-value} = 2 \times P\left(\frac{Y}{150} < \frac{58}{150}\right)$

$= 2 \times P\left(Z < \frac{\frac{58}{150} - 0.4}{\sqrt{\frac{0.4 \times 0.6}{150}}}\right)$

$= 2 \times P(Z < -0.3333)$

$= 2 \times 0.369441$

$= 0.738883$

> 0.05

So we don't have evidence to reject H_0.

We do not have evidence to suggest that the percentage of home owners in favour of the proposals is not 40%.

21. parametric tests require knowing a parameter such as mean or standard deviation of a theoretical distribution. eg. Z-test, t-test,

non-parametric tests make few assumptions about underlying populations and often only use rank order of values rather than numerical values. eg. Mann-Whitney, Wilcoxon tests.

a) we have single sample test for median → Wilcoxon Signed Rank Tests

H_0 : population median, $\eta = 50$
H_1 : population median, $\eta \neq 50$
Assume H_0 to be true
$\alpha = 5\%$
two tail test

Scores	9	11	50	54	58	69	76	91	95
median	50	50	50	50	50	50	50	50	50
s-m	-41	-39	0	4	8	19	26	41	45
|s-m|	41	39		4	8	19	26	41	45
rank	6·5	5.		.1	2	3	4.	6·5	8.

$W_+ = 1+2+3+4+6{\cdot}5+8 = 24{\cdot}5$
$W_- = 6{\cdot}5+5 = 11{\cdot}5$

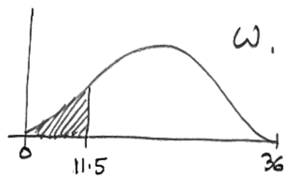
$\Big\}\; 11{\cdot}5 + 24{\cdot}5 = 36 = \frac{1}{2} \times 8 \times 9$ ✓check

$W = \min(W_-, W_+) = 11{\cdot}5$

we have $n=8$

from tables

$P(W_8 \leq 5) = 0{\cdot}05$
$P(W_8 \leq 3) = 0{\cdot}025$



so $P(W_8 \leq 11{\cdot}5) > 0{\cdot}10$ so we are not in critical region

Hence we have no evidence to reject H_0.

We do not have evidence to suggest that the median score is not 50.

cont./

21 b) we can summarise the information, thus:

<50	$=50$	>50
51	19	30

However, we are not able to continue with a Wilcoxon Test as it would require us to know the values of the 51 scores that are below 50, as well as the 30 scores that are above 50.

These scores are needed in order to rank the absolute differences from 50.

The test that's required here is a "Sign Test" which is not part of the AH Statistics course.

22. a) X = survival time in days.

$X \sim N(\mu, \sigma^2)$ by assumption provided.

$H_0: \mu = 400$

$H_1: \mu > 400$

Assume H_0 to be true

$\alpha = 5\%$

one tail test

we estimate σ with $s_{n-1} = 160.042$, and as $n = 10$ is small, we use a t-test

we also know $\bar{x} = 451.3$

so $X \sim N(400, \sigma^2)$

$\bar{X} \sim N(400, \frac{\sigma^2}{10})$ where $\bar{X}$ = mean time of sample of 10, giving $\frac{\bar{X} - 400}{\sqrt{\frac{\sigma^2}{10}}} \sim N(0, 1^2)$

so $t = \frac{\bar{X} - 400}{\sqrt{\frac{s^2}{10}}} \sim t_9$

$$\text{p-value} = P\left(t_9 > \frac{451.3 - 400}{\sqrt{\frac{160.042^2}{10}}}\right)$$

$= P(t_9 > 1.01364)$

$= 0.168614$

> 0.05 so do not reject H_0.

We do not have evidence to suggest that the mean survival time is > 400 days.

b) if we take $\sigma = 150$, we would perform a z-test not a t-test

so $\bar{X} \sim N(400, \frac{150^2}{10})$ giving p-value $= P(\bar{X} > 451.3)$

$$= P\left(Z > \frac{451.3 - 400}{\sqrt{\frac{150^2}{10}}}\right)$$

$= P(Z > 1.0815)$

$= 0.139738$

> 0.05

so no change to logical conclusion.

24 null hypothesis = an assertion that a parameter in a statistical model takes a particular value, and is assumed true until experimental evidence suggests otherwise

critical region = H_0 is rejected when a calculated value of the test statistic lies within this region

test statistic = a function of a sample of observations which provide a basis for testing the validity of H_0

[see CIMT Further Statistics p37/38 for more definitions of key terms]

40 jackets

size	1	2	3	4	5
f	5	8	15	8	4.

$\Sigma f = 40.$

X = no. who have size 3.

$X \sim B(40, p)$

$H_0: p = 0.40$

$H_1: p \neq 0.40$

Assume H_0 to be true

$\alpha = 5\%$

two tail test

$X \sim Bin(40, 0.40)$

$P(X \leq 15) = 0.44022$

$p\text{-value} = 2 \times 0.44022$
$= 0.88044$
> 0.05

So, no evidence to reject H_0

We do not have evidence to suggest that the percentage of employees who require size 3 is not 40%.

/cont.

24 b) We perform a Wilcoxon single sample test on median

we have 15 values that equal 3, so they will give zero differences

therefore, we have $n = 25$

H_0: population median, $\eta = 3$

H_1: population median, $\eta \neq 3$.

Assume H_0 to be true

$\alpha = 5\%$

two tailed test

scores	1	1	1	1	1	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	5	5	5	5
median	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
s−m	−2	−2	−2	−2	−2	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	1	1	2	2	2	2
|s−m|	2	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2
rank			21											8.5									21		

$$\text{so } W_+ = 8 \times 8.5 + 4 \times 21 = 152$$
$$W_- = 5 \times 21 + 8 \times 8.5 = 173.$$

$152 + 173 = 325 = \frac{1}{2} \times 25 \times 26$ ✓check

$$W = \min(W_-, W_+) = 152$$

we have $n = 25$, so we need to perform a Normal approximation

$$E(W) = \frac{1}{4} \times 25 \times 26 = \frac{325}{2}$$
$$V(W) = \frac{1}{24} \times 25 \times 26 \times 51 = \frac{5525}{4}$$

$$\text{so } W \approx N\left(\frac{325}{2}, \frac{5525}{4}\right)$$

$$\text{so p-value} = 2 \times P(W \leq 152)$$
$$= 2 \times P\left(Z \leq \frac{152.5 - \frac{325}{2}}{\sqrt{\frac{5525}{4}}}\right) \quad \text{by c.c.}$$
$$= 2 \times P(Z \leq -0.269069)$$
$$= 2 \times 0.393938 \quad \text{from normCdf}(-9E99, -0.269069)$$
$$= 0.787877$$
$$> 0.05$$

So we do not have evidence to reject H_0

We do not have evidence to suggest that the population median size is not 3.