CIMT Further Statistics p41 Ex 3A

1. $n = 13$

X = length of baby

we shall assume X is normally distributed

$X \sim N(\mu, \sigma^2)$

$H_0 : \mu = 50$

$H_1 : \mu \neq 50$

Assume H_0 to be true

$\alpha = 5\%$

two-tail test

$X \sim N(50, \sigma^2)$

let $\bar{X}$ = sample mean length of 13 babies

$\bar{X} \sim N(50, \frac{\sigma^2}{13})$

$$\frac{\bar{X} - 50}{\sqrt{\frac{\sigma^2}{13}}} \sim N(0, 1^2)$$

we estimate σ with $s_{n-1} = 3.12558$ (from TI-Nspire 1 var stats)

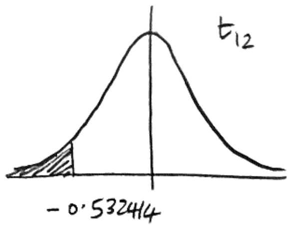
as we have small sample size, and we're estimating σ with s_{n-1}, we have t_{n-1}

$$\frac{\bar{X} - 50}{\sqrt{\frac{s^2}{13}}} \sim t_{12}$$

test statistic, $t = \dfrac{\bar{x} - 50}{\sqrt{\frac{s^2}{13}}}$

$= \dfrac{49.5385 - 50}{\sqrt{\frac{3.12558^2}{13}}}$

$= -0.532414$



p-value $= 2 \times P(t_{12} < -0.532414)$

$= 2 \times 0.302079$

$= 0.604158$

$\gg 0.05$

so we do not have evidence to reject H_0

we do not have evidence to suggest that the mean baby length is not 50cm

[Check: TI-Nspire > Menu > Statistics > t test > Data > $\mu_0 = 50$ > list = length $\boxed{\text{OK}}$]

2. $n=12$

let X = mass of one steel ingot

$X \sim N(\mu, \sigma^2)$ by provided assumption of normality

$H_0: \mu = 25$

$H_1: \mu > 25$

Assume H_0 to be true

$\alpha = 5\%$

1 tail test

we have $s_{n-1} = 2.49739$ and $\bar{x} = 26.03333$

$X \sim N(25, \sigma^2)$

let $\bar{X}$ = mean mass of ignots from sample of size 12

$\bar{X} \sim N(25, \frac{\sigma^2}{12})$

$$\frac{\bar{X} - 25}{\sqrt{\frac{\sigma^2}{12}}} \sim N(0, 1^2)$$

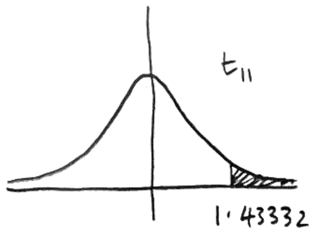
we estimate σ with s_{n-1}, so we use t_{n-1} distribution

$$\frac{\bar{X} - 25}{\sqrt{\frac{s^2}{12}}} \sim t_{11}$$

test statistic, $t = \dfrac{\bar{x} - 25}{\sqrt{\frac{s^2}{12}}}$

$$= \frac{26.03333 - 25}{\sqrt{\frac{2.49739^2}{12}}}$$

$$= 1.43332$$



p-value $= P(t_{11} > 1.43332)$

$= 0.089785$

> 0.05

so we do not have evidence to reject H_0

we do not have evidence to suggest that the mean mass of steel ingots is greater than 25kg.

3. $n=14$

X = milk yield for a cow

we shall assume X to be normally distributed

$X \sim N(\mu, \sigma^2)$

$H_0 : \mu = 120$

$H_1 : \mu > 120$

Assume H_0 to be true

$\alpha = 5\%$

one-tail test

$X \sim N(120, \sigma^2)$

let $\bar{X}$ = mean milk yield for 14 cows, we have $\bar{x} = 138.279$

$s_{n-1} = 24.58$

$\bar{X} \sim N\left(120, \frac{\sigma^2}{14}\right)$

$$\frac{\bar{X}-120}{\sqrt{\frac{\sigma^2}{14}}} \sim N(0, 1^2)$$

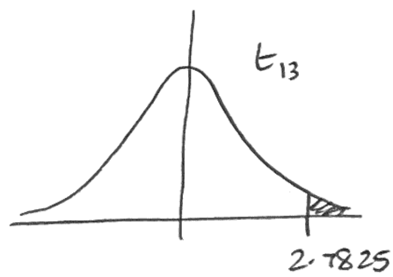
we estimate σ with s_{n-1}, so we use t_{n-1} distribution

$$\frac{\bar{X}-120}{\sqrt{\frac{s^2}{14}}} \sim t_{13}$$

test statistic, $t = \dfrac{\bar{x}-120}{\sqrt{\frac{s^2}{14}}}$

$$= \frac{138.279-120}{\sqrt{\frac{24.58^2}{14}}}$$

$$= 2.7825$$



p-value $= P(t_{13} > 2.7825)$

$= 0.007771$

< 0.05

so we have evidence to reject H_0 and we conclude that the mean milk yield is greater than 120 kg.

4. $n=15$

X = time taken to assemble a flask, in seconds

we assume $X \sim N(\mu, \sigma^2)$

$H_0 : \mu = 120$

$H_1 : \mu < 120$

Assume H_0 to be true

$\alpha = 5\%$

one-tail test

$X \sim N(120, \sigma^2)$

let $\bar{X}$ = mean time taken for a sample of 15 flasks

$\bar{X} \sim N\left(120, \frac{\sigma^2}{15}\right)$ we have $\bar{x} = 116.107$

$s_{n-1} = 16.0951$

$$\frac{\bar{X} - 120}{\sqrt{\frac{\sigma^2}{15}}} \sim N(0, 1^2)$$

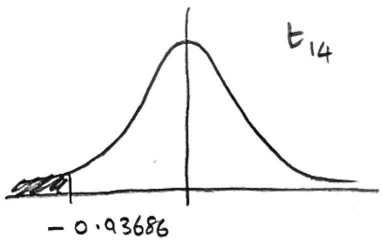
we estimate σ with s_{n-1}, so we use t_{n-1}

$$\frac{\bar{X} - 120}{\sqrt{\frac{s^2}{15}}} \sim t_{14}$$

test statistic, $t = \dfrac{\bar{x} - 120}{\sqrt{\frac{s^2}{15}}}$

$= \dfrac{116.107 - 120}{\sqrt{\frac{16.0951^2}{15}}}$

$= -0.93686$



p-value $= P(t_{14} < -0.93686)$

$= 0.182356$

> 0.05

so we do not have evidence to reject H_0

we do not have evidence to suggest that the mean time taken to assemble a flask is less than 120 seconds.

5. $n=11$

X = percentage extract recovered

we assume X is normally distributed

$X \sim N(\mu, \sigma^2)$

$H_0 : \mu = 95$

$H_1 : \mu \neq 95$

Assume H_0 to be true

$\alpha = 5\%$

two tailed test

$X \sim N(95, \sigma^2)$

let $\bar{X}$ = mean percentage extract recovered from a sample of size 11

$\bar{X} \sim N(95, \frac{\sigma^2}{11})$ we have $\bar{x} = 94.0455$

$s_{n-1} = 1.34117$

$$\frac{\bar{X} - 95}{\sqrt{\frac{\sigma^2}{11}}} \sim N(0, 1^2)$$

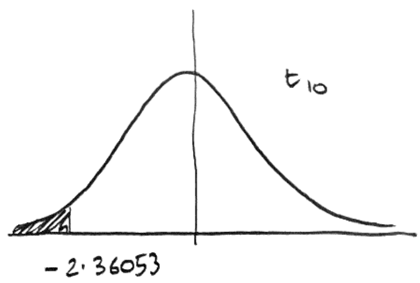
we estimate σ with s_{n-1}, so we use t_{n-1}

$$\frac{\bar{X} - 95}{\sqrt{\frac{S^2}{11}}} \sim t_{10}$$

$$\text{test statistic}, \ t = \frac{\bar{x} - 95}{\sqrt{\frac{s_{n-1}^2}{11}}}$$

$$= \frac{94.0455 - 95}{\sqrt{\frac{1.34117^2}{11}}}$$

$$= -2.36053$$



$$p\text{-value} = 2 \times P(t_{10} < -2.36053)$$
$$= 2 \times 0.019958$$
$$= 0.039917$$
$$< 0.05$$

so we have evidence to reject H_0 and conclude that the mean extract recovered is not 95%

(we conjecture that it is less than 95%)

6. $n=9$

X = time to assemble component, in minutes

we assume X to be normally distributed

$X \sim N(\mu, \sigma^2)$

$H_0 : \mu = 42$

$H_1 : \mu < 42$

Assume H_0 to be true

$\alpha = 5\%$

One tail test

$X \sim N(42, \sigma^2)$

let $\bar{X}$ = mean time to assemble component, from sample of size 9

$\bar{X} \sim N\left(42, \frac{\sigma^2}{9}\right)$ we have $\bar{x} = 36.5556$

$s_{n-1} = 6.65415$

$$\frac{\bar{X} - 42}{\sqrt{\frac{\sigma^2}{9}}} \sim N(0, 1^2)$$

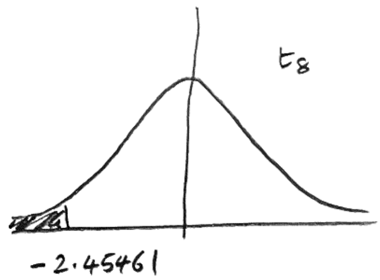
we estimate σ with s_{n-1}, so we use t_{n-1}

$$\frac{\bar{X} - 42}{\sqrt{\frac{s^2}{9}}} \sim t_8$$

test statistic, $t = \dfrac{\bar{x} - 42}{\sqrt{\frac{s^2}{9}}}$

$$= \frac{36.5556 - 42}{\sqrt{\frac{6.65415^2}{9}}}$$

$$= -2.45461$$



p-value $= P(t_8 < -2.45461)$

$= 0.019825$

< 0.05

so we have evidence to reject H_0 and conclude that the mean assembly time of the new method is less than 42 minutes.