

1. we have four  $5 \times 5 \text{ m}^2$  samples, with 2 small faults.

$X$  = no. faults per  $100 \text{ m}^2$

$X \sim \text{Po}(\lambda)$

$$H_0: \lambda = 4 \times 0.925 = 3.7$$

$$H_1: \lambda < 3.7$$

Assume  $H_0$  to be true.

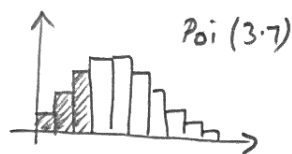
$$\alpha = 5\%$$

one-tail test

$$p\text{-value} = P(X \leq 2)$$

$$= 0.285433$$

$$> 0.05$$



so we do not have evidence to reject  $H_0$  and so we conclude that the mean number of faults has not been reduced from 0.925 per  $25 \text{ m}^2$ .

2. 8 parasites found on 3 fish

$X$  = total number of parasites on 3 fish.

$$X \sim \text{Po}(\lambda)$$

$$H_0: \lambda = 3 \times 4 = 12$$

$$H_1: \lambda \neq 12$$

Assume  $H_0$  is true

$$\alpha = 5\%$$

two tail test

as poisson distribution is not symmetrical, we need to be careful about calculating p-values.

$$P(X \leq 8) = 0.155028 \quad \text{from } \text{poissCDF}(12, 0, 8)$$

This is clearly in excess of 2.5%, and 5%, so we can confidently conclude that we cannot reject  $H_0$  and thus we do not have evidence that there is not 4 parasites per fish.

3.  $X =$  no. of complaint letters in 150 days.

$$X \sim \text{Po}(\lambda)$$

$$H_0: \lambda = 150 \times 3 = 450$$

$$H_1: \lambda < 450$$

we assume  $H_0$  to be true.

$$\alpha = 1\%$$

one-tail test

$$\text{so } X \sim \text{Po}(450)$$

$$P(X \leq 407) = 0.021263 \\ < 0.05$$

OR

we can approx with normal, as  $\lambda > 10$

let  $Y =$  normal approx to  $X$

$$Y \sim N(450, 450)$$

$$P(X \leq 407) = P(Y < 407.5) \quad \text{by c.c.}$$

$$= P\left(Z < \frac{407.5 - 450}{\sqrt{450}}\right)$$

$$= P(Z < -2.00347)$$

$$= 0.022563$$

$$< 0.05$$

either way, we have evidence to reject  $H_0$  and conclude that there are fewer than 3 letters of complaint per day.

Assumption needed is that the number of letters per day is independent of other days  
i.e. there has not been a single incident that triggered a lot of complaints, for example.

4.  $X$  = no. of breakages per 8 weeks.

$$X \sim \text{Po}(\lambda)$$

$$H_0: \lambda = 27 \times 8 = 216$$

$$H_1: \lambda < 216$$

Assume  $H_0$  to be true

$$\alpha = 5\%$$

one-tail test

we observed a total of  $23 + 19 + \dots + 22 + 26 = 184$  breakages

$$P(X \leq 184) = 0.014364 \text{ from Poiss Cdf}(216, 0, 184) \\ < 0.05$$

OR

let  $Y$  be normal approx to  $X$  as  $\lambda > 10$

$$Y \sim N(216, 216)$$

$$P(X \leq 184) = P(Y < 184.5) \text{ by c.c.}$$

$$= P\left(Z < \frac{184.5 - 216}{\sqrt{216}}\right)$$

$$= P(Z < -2.1433)$$

$$= 0.016044$$

$$< 0.05$$

either way, we will reject  $H_0$  and conclude that the manager's announcement has been heeded, resulting in the mean no. of breakages per week to be less than 27.