

1.

score	1	2	3	4
O_i	12	15	19	22
E_i	17	17	17	17

$\sum O_i = 68$

H_0 : scores distributed as a discrete uniform distribution, $U(4)$.

H_1 : scores are not $U(4)$, and thus die is biased.

Assume H_0 to be true.

$\alpha = 5\%$, one tail test

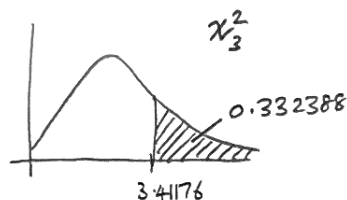
under H_0 , E_i are all $\frac{68}{4} = 17$

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

$$= 3.41176$$

$$df = 4 - 1 = 3.$$

$$\therefore \chi^2 \sim \chi^2_3$$



$$\therefore P(\chi^2 \geq 3.41176) = 0.332388$$

$$> 0.05$$

so we do not reject H_0

we do not have evidence to suggest that the scores are not uniformly distributed.

this suggests that the die is fair.

2.

Score	1	2	3	4	5	6
A_o	17	20	29	20	18	16
f_e	20	20	20	20	20	20.

H_0 : scores are distributed as discrete $U(6)$

H_1 : scores are not $\sim U(6)$

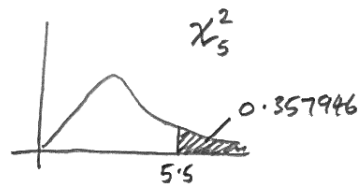
Assume H_0 to be true

$\alpha = 5\%$, one-tailed test

$$df = 5$$

$$\begin{aligned} \chi^2 &= \sum \frac{(O_i - E_i)^2}{E_i} \\ &= 5.5 \end{aligned}$$

$$\text{now } \chi^2 \sim \chi^2_5$$



$$\begin{aligned} P(\chi^2 \geq 5.5) &= 0.357946 \\ &> 0.05 \end{aligned}$$

so we do not reject H_0

we do not have evidence to suggest that the scores are not uniformly distributed

this suggests that the die is not biased.

3.

Day	M	Tu	Wed	Thr	Fri
f_o	125	88	85	94	108
f_e	100	100	100	100	100

H_0 : absences uniformly distributed, with discrete $U(5)$

H_1 : absences dependent on day of week, and not distributed $U(5)$

Assume H_0 to be true

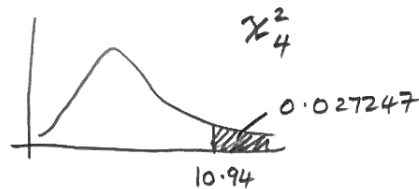
$\alpha = 5\%$, one-tailed test

under H_0 , $f_e = 100$, $df = 4$

$$X^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

$$= 10.94$$

$$X^2 \sim \chi^2_4$$



$$P(X^2 > 10.94) = 0.027247$$

$$< 0.05$$

so we reject H_0

so we have evidence to suggest that absences are dependent on the day of the week.

4.

Day M Tu W Th F Sa Su

f_o 60 54 48 53 53 75 77 $\sum f_o = 420$

f_e 60 60 60 60 60 60 60

H_0 : car accidents are uniformly distributed discretely as $u(7)$

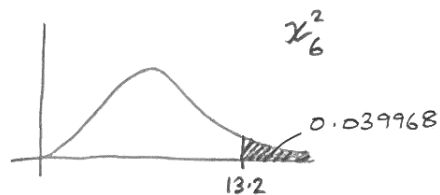
H_1 : car accidents are not uniformly distributed as $u(7)$

Assume H_0 to be true.

$\alpha = 5\%$ one-tailed test

under H_0 , $f_e = 60$ $df, \nu = 7 - 1 = 6$

$$\begin{aligned} \chi^2 &= \sum \frac{(f_o - f_e)^2}{f_e} \\ &= 13.2 \end{aligned}$$



$$\begin{aligned} P(\chi^2 > 13.2) &= 0.039968 \\ &< 0.05 \end{aligned}$$

Hence we have evidence to reject H_0 and conclude that the data are not a sample from a uniform distribution.

5.

Doors	N	S	E	W	
f_o	327	402	351	380	$\sum f_o = 1460.$
f_e	365	365	365	365	

H_0 : customers are distributed as discrete $U(4)$

H_1 : customers are not distributed as $U(4)$

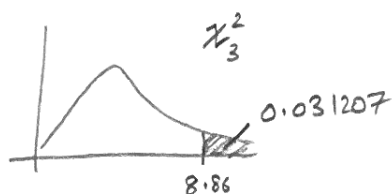
Assume H_0 to be true.

under H_0 , $f_e = 365$

$\alpha = 5\%$, one tailed test,

$$df = 4 - 1 = 3.$$

$$\begin{aligned} \chi^2 &= \sum \frac{(f_o - f_e)^2}{f_e} \\ &= 8.86027 \end{aligned}$$



$$\begin{aligned} \text{so } P(\chi^2 > 8.86) &= 0.031207 \\ &< 0.05 \end{aligned}$$

Hence we have evidence to reject H_0 and conclude that the four exits are not equally used. (It would appear as though most traffic uses the South and West doors).

6.

Blood Type	O	A	B	AB	
f_o	87	59	20	4	$\Sigma f_o = 170.$
p_e	0.49	0.38	0.09	0.04	expected ratios $\frac{49:38:9:4}{\approx 100}$
f_e	83.3	64.6	15.3	6.8	

H_0 : blood type proportions are similar to general population

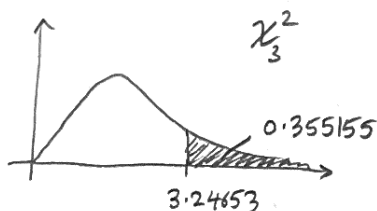
H_1 : blood type proportions are different to general population

Assume H_0 to be true.

$\alpha = 5\%$, one-tail test

$$df = 4 - 1 = 3.$$

$$\begin{aligned} \chi^2 &= \sum \frac{(f_o - f_e)^2}{f_e} \\ &= 3.24653 \end{aligned}$$



$$\text{so } P(\chi^2 > 3.24653) = 0.355155$$

> 0.05 , so we do not reject H_0 .

Hence we do not have evidence to suggest that the small community has different proportions of blood type to those in the general population.