Normal Approximations for
Mann-Whitney and Wilcoxon Signed Rank Sum Hypothesis Tests

When we have sample sizes that exceed 20, the data tables can no longer be used.

In these cases, we can approximate the distributions of rank rums, W, with the Normal Distribution.

For Wilcoxon tests, we use the following to approximate the parameters of W:

$$\mathrm{E}(W)=\frac{1}{4}n(n+1) \text{ and } \mathrm{V}(W)=\frac{1}{24}n(n+1)(2n+1)$$

For Mann-Whitney tests, we use the following to approximate the parameters of W:

$$\mathrm{E}(W)=\frac{1}{2}m(m+n+1) \text{ and } \mathrm{V}(W)=\frac{1}{12}mn(m+n+1)$$

Each of the questions in this document provides you with the full set of raw data as well as the minimum summary statistic(s) required to complete the appropriate hypothesis test.

The TI-Nspire file called 'Normal Approx to MW and Wilcoxon.tns' contains all the raw data.

You should practice completing questions using both the raw data <u>and</u> using just the summary statistic(s).

In addition, you should <u>always</u> generate a display of the raw data to obtain a subjective impression of the situation, before employing your chosen hypothesis testing procedure.

Full worked solutions are provided after each question. Lovely.

Data Sets and Questions sourced from the following online publications:

- *'Chapter 25 Non-parametric Tests', published 25 Aug 2008, FREE013-Moore.*
  http://www.math.utah.edu/~firas/1070/bps5e_chapter25.pdf
- *'Real Statistics Using Excel', wordpress website*
  http://www.real-statistics.com/free-download/
- *'Mann-Whitney U Test' for 'Farm Boys and Town Boys' question*
  http://www.statsdirect.co.uk/help/content/nonparametric_methods/mann_whitney.htm
- *'Research Skills, Graham Hole: Nonparametric tests with large sample sizes.' for 'Ant and Dec' question.*
  http://users.sussex.ac.uk/~grahamh/RM1web/Wilcoxon%20Large%20N%202009.pdf

Q1. Farm Boys vs Town Boys

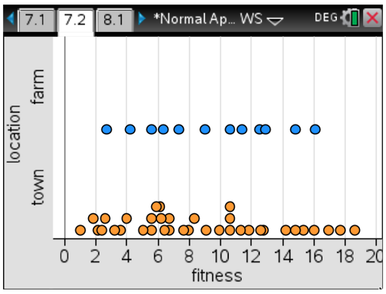
The following data sets represent fitness scores from two groups of boys of the same age, those from homes in the town and those from farm homes.

Farm Boys	14.8	7.3	5.6	6.3	9	4.2	10.6	12.5	12.9	16.1	11.4	2.7
Town Boys	12.7	14.2	12.6	2.1	17.7	11.8	16.9	7.9	16	10.6	5.6	5.6
	7.6	11.3	8.3	6.7	3.6	1	2.4	6.4	9.1	6.7	18.6	3.2
	6.2	6.1	15.3	10.6	1.8	5.9	9.9	10.6	14.8	5	2.6	4

Analyse the data and report your conclusions.

Summary Statistic:
W_{TOWN}=855

Full Worked Solution to Farm Boys vs Town Boys



Data is not paired.
If we assume that the distributions of the fitness scores have similar shape and spread, then we can perform a Mann-Whitney test.
The dot plots show that this assumption is plausible.
Farm boys are not obviously more or less fit, so we shall perform a two-tailed test on the sample medians.

We should also assume that these groups of boys are independent and that they represent at least hypothetical random samples of the sub-populations they represent.

H_0: population median$_{FARM}$ = population median$_{TOWN}$
H_1: population median$_{FARM} \neq$ population median$_{TOWN}$
α=5%. Two tailed test.
Assume H_0 is true.

Ranking the data gives:

T	T	T	T	T	F	T	T	T	F	T	T	T	F	T	T	T	F	T	T	T	F	T	T
1	1.8	2.1	2.4	2.6	2.7	3.2	3.6	4	4.2	5	5.6	5.6	5.6	5.9	6.1	6.2	6.3	6.4	6.7	6.7	7.3	7.6	7.9
1	2	3	4	5	6	7	8	9	10	11	13	13	13	15	16	17	18	19	20.5	20.5	22	23	24

T	F	T	T	T	T	T	F	T	F	T	F	T	T	F	T	T	F	T	T	F	T	T	T
8.3	9	9.1	9.9	10.6	10.6	10.6	10.6	11.3	11.4	11.8	12.5	12.6	12.7	12.9	14.2	14.8	14.8	15.3	16	16.1	16.9	17.7	18.6
25	26	27	28	30.5	30.5	30.5	30.5	33	34	35	36	37	38	39	40	41.5	41.5	43	44	45	46	47	48

We would reject H_0 for either large or small values of either W_{TOWN} or W_{FARM}
W_{FARM}=321 (m=12) and W_{TOWN}=855 (n=36)
We focus on W_{FARM} as it has the smallest sample size, m=12

As n>20, we approximate W_{FARM} with W=normal approximation to W_{FARM},

$$W \sim N\left(\tfrac{1}{2}12(12+36+1), \tfrac{1}{12}12\times 36(12+36+1)\right)$$
$$W \sim N(294, 1764)$$

So, we want to know $P(W_{FARM} \geq 321)$, as 321 is greater than the mean of 294.

$$\begin{aligned} P(W_{FARM} \geqslant 321) &\approx P(W > 320.5) \text{ by continuity correction} \\ &= P\left(Z > \frac{320.5-294}{\sqrt{1764}}\right) \\ &= P(Z > 0.630952...) \\ &= 0.264036.. \text{ from normcdf}(0.630952.., 9E99) \\ &\approx 0.2640\ (4dp) \end{aligned}$$

Now as we have a two tail test, *either* we compare 0.2640 to 0.025 (half the alpha value)
or we compare 2×0.2640 to 0.05 (the alpha value)

We should always compare a p-value to the alpha value, so here

p-value = $2\times P(W_{FARM} \geq 321) \approx 0.5281 > 0.05$

Hence we are not in the critical region, and we have no reason to reject the null hypothesis.
We conclude that the population median fitness of town boys does not appear to be different to that for farm boys.

Q2. Do Good Smells bring Good Business?

An experiment asked whether background aromas in a restaurant encourage customers to stay longer and spend more. The data on amount spent (in euros) has been collected.

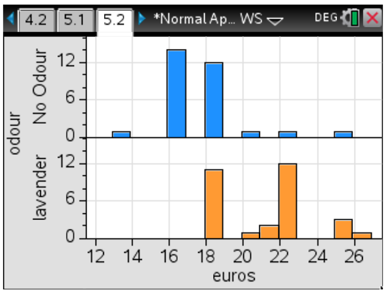
No Odour	15.90	18.50	15.90	18.50	18.50	21.90	15.90	15.90	15.90	15.90	15.90	18.50	18.50	18.50	20.50
Lavender	21.90	18.50	22.30	21.90	18.50	24.90	18.50	22.50	21.50	21.90	21.50	18.50	25.50	18.50	18.50
No Odour	18.50	18.50	15.90	15.90	15.90	18.50	18.50	15.90	18.50	15.90	18.50	15.90	25.50	12.90	15.90
Lavender	21.90	18.50	18.50	24.90	21.90	25.90	21.90	18.50	18.50	22.80	18.50	21.90	20.70	21.90	22.50

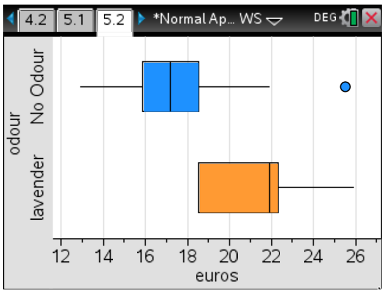
Is there significant evidence that the lavender odour encourages customers to spend more?

Summary Statistic:

$W_{LAVENDER}=1241.5$

Full Worked Solution to Do Good Smells bring Good Business





Data is not paired.
If we assume that the distributions of the money paid have similar shape and spread, then we can perform a Mann-Whitney test.
The dot plots show that this assumption is (just!) plausible.

We are asked to establish whether lavender induces customers to spend more, so we shall perform a one-tailed test. By comparison of the boxplots, we expect the null hypothesis to be rejected. Let's see if it is.....

We should also assume that these groups of customers are independent and that they represent at least hypothetical random samples of the sub-populations they represent.

H_0: population median$_{\text{LAVENDER}}$=population median$_{\text{NONE}}$ α=5%. One tailed test.
H_1: population median$_{\text{LAVENDER}}$>population median$_{\text{NONE}}$ Assume H_0 is true.

Ranking the data gives:

N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
12.90	15.90	15.90	15.90	15.90	15.90	15.90	15.90	15.90	15.90	15.90	15.90	15.90	15.90	15.90	18.50	18.50	18.50	18.50	18.50
1	8.5	8.5	8.5	8.5	8.5	8.5	8.5	8.5	8.5	8.5	8.5	8.5	8.5	8.5	27	27	27	27	27

N	N	N	N	N	N	N	L	L	L	L	L	L	L	L	L	L	L	N	L
18.50	18.50	18.50	18.50	18.50	18.50	18.50	18.50	18.50	18.50	18.50	18.50	18.50	18.50	18.50	18.50	18.50	18.50	20.50	20.70
27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	39	40

L	L	N	L	L	L	L	L	L	L	L	L	L	L	L	L	L	N	L	L
21.50	21.50	21.90	21.90	21.90	21.90	21.90	21.90	21.90	21.90	21.90	22.30	22.50	22.50	22.80	24.90	24.90	25.50	25.50	25.90
41.5	41.5	47	47	47	47	47	47	47	47	47	52	53.5	53.5	55	56.5	56.5	58.5	58.5	60

We would reject H_0 for either large values of either W_{LAVENDER} or small values of W_{NONE}
If we use the summary statistic, then

$$W_{NONE} = \tfrac{1}{2} \times 60 \times 61 - 1241.5 = 1830 - 1241.5 = 588.5$$

W_{LAVENDER}=1241.5 (m=30) and W_{NONE}=588.5 (n=30)
We can focus on either W_{NONE} or W_{LAVENDER} as the samples are equal size.
Let's use W_{LAVENDER} as it was provided to us.

As m,n>20, we approximate W_{LAVENDER} with W=normal approximation to W_{LAVENDER},

$$W \sim N\left(\tfrac{1}{2}30(30+30+1), \tfrac{1}{12}30\times 30(30+30+1)\right)$$
$$W \sim N(915, 4575)$$

So, we want to know P(W_{LAVENDER}≥1241.5), as we reject H_0 for large values of W_{LAVENDER}

$$\begin{aligned} P(W_{LAVENDER} \geqslant 1241.5) &\approx P(W > 1241) \ \textit{by continuity correction} \\ &= P\left(Z > \frac{1241-915}{\sqrt{4575}}\right) \\ &= P(Z > 4.81972...) \\ &= 0.000000719737.. \ \textit{from normcdf}(4.81972.., 9E99) \\ &\approx 0.0000\,(4dp) \end{aligned}$$

We are clearly in the 5% tail, and thus we can confidently reject the null hypothesis and conclude from this sample that the Good Smell of Lavender does generate more median income for the restaurant.

Q3. Turning Right Versus Turning Left

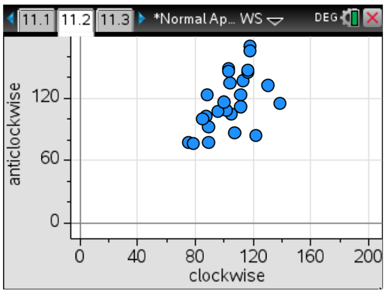
Contains data from a student project that investigated whether right-handed people can turn a handle faster clockwise than they can anti-clockwise.

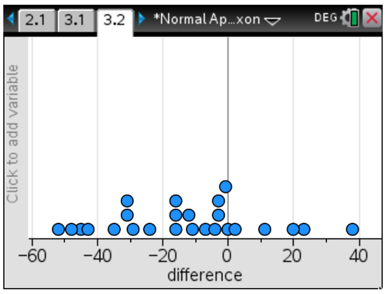
Clockwise	113	105	130	101	138	118	87	116	75	96	122	103	116	107	118	103	111	104	111	89	78	100	89	85	88
Anti-Clockwise	137	105	133	108	115	170	103	145	78	107	84	148	147	87	166	146	123	135	112	93	76	116	78	101	123

Describe what the data show, then state hypotheses and do a test.
Report your conclusions carefully.

Summary Statistic:
W=56.5

Full Worked Solution to "Turning Right versus Turning Left"





Data is paired.
If we assume that the distributions of the differences of the turning scores are symmetrical (supported by the dot plots) then we can do a Wilcoxon Signed Rank Sum test.

We are asked to establish whether right handed people turn handles <u>faster</u> clockwise compared to anti-clockwise. Hence this will be a one-tailed test.

Subjective impression from the dotplots suggests that if low numbers mean turning faster, that clockwise turning is quicker

H_0: population $\text{median}_{\text{DIFFERENCE}}=0$ where difference = clockwise-anticlockwise
H_1: population $\text{median}_{\text{DIFFERENCE}}<0$ (ie right handers turn clockwise faster than anticlockwise)
$\alpha=5\%$. One tailed test.
Assume H_0 is true.

Ranking the data gives:

Subject	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Clockwise	113	105	130	101	138	118	87	116	75	96	122	103	116	107	118	103	111	104	111	89	78	100	89	85	88
Anti-Clockwise	137	105	133	108	115	170	103	145	78	107	84	148	147	87	166	146	123	135	112	93	76	116	78	101	123
C-AC	-24	0	-3	-7	23	-52	-16	-29	-3	-11	38	-45	-31	20	-48	-43	-12	-31	-1	-4	2	-16	11	-16	-35
|C-AC|	24	0	3	7	23	52	16	29	3	11	38	45	31	20	48	43	12	31	1	4	2	16	11	16	35
rank	15		3.5	6	14	24	11	16	3.5	7.5	20	22	17.5	13	23	21	9	17.5	1	5	2	11	7.5	11	19
Wpos					14						20			13							2		7.5		

We rejected the tied results that gave an absolute difference of 0, leaving us with 24 pairs.
So, $W_{POS}=56.5$ and $Wneg=\frac{1}{2}\times 24\times 25-56.5=300-56.5=243.5$
We focus on the minimum, $W_{pos}=56.5$

We would reject H_0 for small values of W_{pos}

As n>20, we approximate W_{POS} with W=normal approximation to W_{POS},

$$W\sim N\left(\tfrac{1}{4}24(24+1),\tfrac{1}{24}24(24+1)(2\times 24+1)\right)$$
$$W\sim N(150,1225)$$
$$W\sim N(150,35^2)$$

So, we want to know $P(W_{POS}\le 56.5)$, as we reject H_0 for small values of W_{POS}

$$\begin{aligned}P(W_{POS}\leqslant 56.5) &\approx P(W<57)\ \textit{by continuity correction}\\ &=P\left(Z<\frac{57-150}{35}\right)\\ &=P(Z<-2.65714...)\\ &=0.00394..\ \textit{from normcdf}(-9E99,-2.65714)\\ &\approx 0.0039\,(4dp)\end{aligned}$$

We are clearly in the 5% tail, and thus we can confidently reject the null hypothesis and conclude that right handed people turn handles clockwise faster than they turn them anticlockwise, where faster means their median turning time.

Q4. Food Safety

Food sold at outdoor fairs and festivals may be less safe than food sold in restaurants because it is prepared in temporary locations and often by volunteer help. What do people who attend fairs think about the safety of the food served? One study asked this question of people at a number of fairs in the Midwest, USA.

"How often do you think people become sick because of
food they consume prepared at outdoor fairs and festivals?"

The possible responses were:

1 = very rarely
2 = once in a while
3 = often
4 = more often than not
5 = always

Note that the numerical difference between 'very rarely' and 'once in a while' is the same as the difference between 'once in a while' and 'often'. This may not make numerical sense. A rank test only uses the order of these responses, not their actual value. The responses can be arranged in order from least to most concerned about safety, so a rank test makes sense.

The researcher visited 11 different fairs. She stood near the entrance and stopped every 25th adult who passed. Because no personal choice was involved in choosing the subjects, we can reasonably treat the data as coming from a random sample. (As usual, there was some non-response, which could create bias)

In all, 303 people answered the question about fairs, as well as a similar question relating to fast food chains and restaurants. Of these, 196 were women and 107 were men.

The full set of data of opinions about food safety at Fairs, Fast Food Chains and Restaurants is on the next page.

We suspect that women are more concerned about food safety than men.

a) Explain carefully why we cannot answer this question by applying a Wilcoxon rank sum test to the variables 'Fair Safety' and 'Restaurant Safety', or indeed any other pairing of the data.

b) Conduct an appropriate test to establish whether women are indeed more concerned than men, for **only one** of the food outlets of your choosing: Fairs, Fast Food Chains or Restaurants.

Summary Statistics

For Fairs, W_{WOMEN}=31995.5
For Fast Food, W_{WOMEN}=32007.5
For Restaurants, W_{WOMEN}= 32267.5

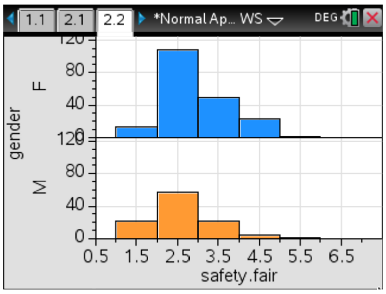
Raw data for Food Safety

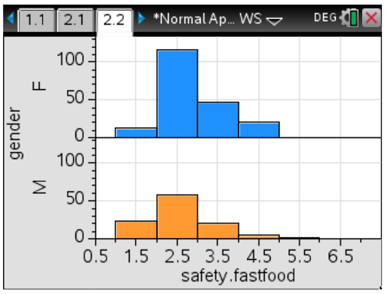
Subject	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Gender	F	F	F	F	F	F	F	F	F	F	F	F	F	F	M	M	M	M	M	M	M	M	M	M	M	F	M	M	M	M
Fair Safety	1	2	2	2	3	2	3	2	2	2	2	2	2	2	2	2	1	2	2	2	2	1	2	2	2	3	2	2	2	1
Fast Food Safety	1	4	2	2	1	2	2	4	3	2	2	2	2	2	2	2	1	2	2	4	2	1	2	1	3	3	3	2	2	1
Restaurant Safety	1	2	2	1	3	2	2	4	2	2	2	1	1	2	2	2	1	2	1	3	2	1	1	1	2	2	2	1	2	1
Subject	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Gender	M	M	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	M	M	F	F	F	F	F	M	F	F
Fair Safety	2	1	2	4	2	2	1	2	2	3	2	2	2	2	1	1	2	2	1	2	1	1	2	2	4	2	3	2	2	3
Fast Food Safety	1	3	3	4	2	2	1	2	2	3	2	2	1	2	2	2	2	4	1	4	1	1	2	2	2	2	2	1	2	2
Restaurant Safety	1	2	2	3	1	2	1	1	1	1	1	2	1	2	1	1	2	1	1	1	1	1	2	2	2	2	2	1	2	2
Subject	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
Gender	F	M	F	F	F	F	F	M	M	M	M	M	M	M	M	M	M	F	F	F	F	F	F	F	M	F	F	M	F	F
Fair Safety	2	3	2	4	2	2	4	1	2	3	1	2	1	3	1	4	1	2	2	2	3	2	2	2	2	2	3	2	3	2
Fast Food Safety	3	2	4	3	2	2	2	2	2	4	1	2	1	2	2	2	1	2	2	3	3	3	2	2	2	2	3	2	3	2
Restaurant Safety	2	3	3	3	2	2	1	1	2	2	1	1	1	2	1	2	1	2	2	1	3	3	2	2	1	1	2	2	2	2
Subject	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
Gender	F	F	F	F	F	F	F	F	F	F	F	F	F	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
Fair Safety	2	3	2	1	1	2	3	2	2	3	4	1	2	3	1	2	2	2	5	4	2	4	2	3	2	3	3	2	2	2
Fast Food Safety	2	1	2	2	3	3	4	2	2	3	4	2	2	1	2	2	2	2	2	2	2	2	1	5	2	3	3	3	2	3
Restaurant Safety	2	1	2	1	2	2	1	2	1	2	3	2	1	5	1	1	1	2	1	2	1	1	1	5	2	1	2	2	1	2
Subject	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150
Gender	F	F	M	F	F	F	F	F	F	F	F	M	M	M	M	M	M	F	F	M	M	F	F	F	F	F	F	F	F	M
Fair Safety	3	3	3	3	2	2	3	2	2	2	2	2	2	2	2	1	1	3	3	1	1	3	2	2	3	3	4	1	2	1
Fast Food Safety	3	2	2	3	2	2	4	2	3	1	2	2	2	2	3	2	4	2	3	1	2	3	2	2	2	2	3	2	2	1
Restaurant Safety	2	2	2	3	2	2	1	2	2	1	2	1	1	2	1	1	2	2	2	1	2	2	2	1	2	2	2	1	1	1
Subject	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
Gender	M	M	M	M	F	F	F	F	F	F	F	F	F	F	F	M	M	F	F	F	F	M	M	M	F	F	F	F	F	M
Fair Safety	3	3	2	1	2	5	2	3	4	2	2	2	4	2	2	2	3	2	3	3	1	3	2	1	4	4	3	2	4	2
Fast Food Safety	2	3	2	1	2	2	2	3	3	2	2	2	2	3	2	3	1	3	3	3	1	2	1	1	3	2	3	2	4	2
Restaurant Safety	2	1	1	1	2	2	2	2	2	2	2	2	1	2	2	3	1	1	2	3	1	2	1	1	2	2	3	2	2	1
Subject	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210
Gender	F	F	F	F	F	F	F	F	F	F	M	M	M	M	M	M	F	M	F	M	F	F	F	F	F	M	F	F	F	F
Fair Safety	2	2	4	3	2	4	4	2	3	3	2	2	3	2	2	2	3	4	4	2	4	2	2	3	3	3	2	1	2	2
Fast Food Safety	2	2	4	2	2	4	2	4	2	3	2	1	3	2	3	2	2	2	3	2	2	4	2	3	2	3	2	2	3	2
Restaurant Safety	2	2	2	1	2	1	2	2	2	2	2	1	3	2	2	2	2	2	2	2	1	4	2	3	2	2	2	1	2	2
Subject	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240
Gender	F	F	M	M	F	F	M	F	F	F	M	F	F	F	F	F	F	F	M	F	F	F	M	F	F	F	F	M	M	F
Fair Safety	2	3	4	2	2	3	3	2	3	3	2	3	2	2	3	1	2	2	1	2	4	4	2	2	2	3	2	3	2	3
Fast Food Safety	2	4	4	2	2	3	2	1	3	2	2	3	2	3	3	1	2	2	1	2	2	2	2	3	2	2	4	3	1	3
Restaurant Safety	2	2	2	2	2	3	2	1	2	2	1	2	2	2	3	1	1	1	1	2	2	2	2	2	2	2	2	3	1	2
Subject	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270
Gender	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	M	M	M	M	M	M	M	M	F
Fair Safety	2	4	2	3	2	2	2	4	2	3	2	2	4	2	2	2	2	3	3	4	1	1	2	3	2	3	2	2	2	2
Fast Food Safety	2	4	2	2	1	2	2	2	2	2	2	3	4	2	2	2	1	1	2	2	3	1	2	2	3	3	2	2	1	2
Restaurant Safety	2	4	2	3	1	2	2	5	2	2	2	1	4	2	2	1	1	1	2	2	2	1	1	1	2	3	2	1	1	2
Subject	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300
Gender	F	F	F	M	M	M	M	F	F	M	F	M	F	F	M	M	F	M	F	F	F	F	F	M	F	F	M	F	F	F
Fair Safety	2	2	2	2	2	3	2	3	3	2	2	2	2	2	2	3	2	2	5	3	2	2	2	3	2	2	3	3	3	3
Fast Food Safety	2	2	2	2	2	2	2	3	2	2	2	3	2	3	3	3	2	3	4	2	3	2	3	2	2	2	2	4	3	2
Restaurant Safety	2	1	1	1	1	3	1	3	2	2	2	2	2	2	2	2	2	3	3	2	2	2	4	2	3	3	2	2	3	2
Subject	301	302	303																											
Gender	F	F	M																											
Fair Safety	4	3	1																											
Fast Food Safety	3	2	2																											
Restaurant Safety	1	2	2																											

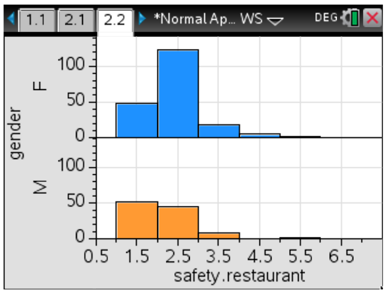
Full Worked Solution to Food Safety

a) The reason that a Wilcoxon Signed Paired Rank Test cannot be used here is that to compare women's views to men's views, we need to pair up one man to one woman. This is not possible as a man and woman are different.

IF we had data on, say, 200 named food outlets and asked men and women about each of those 200 food outlets, then those 200 pieces of paired data could be analysed using a paired test for men's and women's views on each of the food outlets.







b) Data is not paired.
If we assume that the distributions of the ages have similar shape and spread, then we can perform a Mann-Whitney test.
The dot plots for Fairs and Fast Food show that this assumption is plausible.
We are more doubtful about the distribution for Restaurants.
The question asks if women and more concerned than men, so we shall perform a 1-tail test.

Safety of Fair Food (other data tests are on the next page)

H_0: population median$_{WOMEN}$ = population median$_{MEN}$ $\qquad$ α=5%. One tailed test.
H_1: population median$_{WOMEN}$ > population median$_{MEN}$ $\qquad$ Assume H_0 is true.

Ranking the data takes a long time, so we shall use the summary data.

We would reject H_0 for either large values of W_{WOMEN} or small values of W_{MEN}
We need to focus on W_{MEN}, as it has the smallest sample size, m=107
We are given W_{WOMEN}=31995.5 (n=196), and we have 107+196=303 data points, so

$$W_{MEN} = \tfrac{1}{2} \times 303 \times 304 - 31995.5 = 14060.5$$

As m,n>20, we approximate W_{MEN} with W=normal approximation to W_{MEN},

$$W \sim N(\tfrac{1}{2}107(107+196+1), \tfrac{1}{12}107 \times 196(107+196+1))$$
$$W \sim N(16264, \frac{1593872}{3})$$

So, we want to know P(W_{MEN}≤14060.5), as we reject H_0 for small values of W_{MEN}

$$\begin{aligned} P(W_{MEN} \leqslant 14060.5) &\approx P(W < 14061) \textit{ by continuity correction} \\ &= P(Z < \frac{14061-16264}{\sqrt{1593872/3}}) \\ &= P(Z < -3.02238...) \\ &= 0.001254.. \textit{ from normcdf}(-9E99, -3.02238..) \\ &\approx 0.0013(4dp) \end{aligned}$$

Now as we have a one tail test

p-value = P(W_{MEN}≤14060.5) ≈ 0.0013 < 0.05

Hence we are in the critical region, and we have reason to reject the null hypothesis.
We conclude that women are more concerned about food safety at Fairs, than men.

Safety of Fast Food Chains

H_0: population median$_{WOMEN}$=population median$_{MEN}$ α=5%. One tailed test.
H_1: population median$_{WOMEN}$ >population median$_{MEN}$ Assume H_0 is true.

We would reject H_0 for either large values of W_{WOMEN} or small values of W_{MEN}
We need to focus on W_{MEN}, as it has the smallest sample size, m=107
We are given W_{WOMEN}=32007.5 (n=196), and we have 107+196=303 data points, so

$$W_{MEN} = \tfrac{1}{2} \times 303 \times 304 - 32007.5 = 14048.5$$

As m,n>20, we approximate W_{MEN} with W=normal approximation to W_{MEN},

$$W \sim N\left(\tfrac{1}{2}107(107+196+1), \tfrac{1}{12}107 \times 196(107+196+1)\right)$$
$$W \sim N\left(16264, \frac{1593872}{3}\right)$$

So, we want to know $P(W_{MEN} \leq 14048.5)$, as we reject H_0 for small values of W_{MEN}

$$\begin{aligned} P(W_{MEN} \leqslant 14048.5) &\approx P(W < 14049) \ \textit{by continuity correction} \\ &= P\left(Z < \frac{14049-16264}{\sqrt{1593872/3}}\right) \\ &= P(Z < -3.03884...) \\ &= 0.001188.. \ \textit{from normcdf}(-9E99, -3.03884..) \\ &\approx 0.0012\,(4dp) \end{aligned}$$

Now as we have a one tail test

$$\text{p-value} = P(W_{MEN} \leq 14048.5) \approx 0.0012 < 0.05$$

Hence we are in the critical region, and we have reason to reject the null hypothesis.
We conclude that women are more concerned about food safety at Fast Food Chains, than men.

Safety of Restaurants

H_0: population median$_{WOMEN}$=population median$_{MEN}$ α=5%. One tailed test.
H_1: population median$_{WOMEN}$ >population median$_{MEN}$ Assume H_0 is true.

We would reject H_0 for either large values of W_{WOMEN} or small values of W_{MEN}
We need to focus on W_{MEN}, as it has the smallest sample size, m=107
We are given W_{WOMEN}= 32267.5 (n=196), and we have 107+196=303 data points, so

$$W_{MEN} = \tfrac{1}{2} \times 303 \times 304 - 32267.5 = 13788.5$$

As m,n>20, we approximate W_{MEN} with W=normal approximation to W_{MEN},

$$W \sim N\left(\tfrac{1}{2}107(107+196+1), \tfrac{1}{12}107 \times 196(107+196+1)\right)$$
$$W \sim N\left(16264, \frac{1593872}{3}\right)$$

So, we want to know $P(W_{MEN} \leq 13788.5)$, as we reject H_0 for small values of W_{MEN}

$$\begin{aligned} P(W_{MEN} \leqslant 13788.5) &\approx P(W < 13789) \ \textit{by continuity correction} \\ &= P\left(Z < \frac{13789-16264}{\sqrt{1593872/3}}\right) \\ &= P(Z < -3.39554...) \\ &= 0.000343.. \ \textit{from normcdf}(-9E99, -3.39554...) \\ &\approx 0.0003\,(4dp) \end{aligned}$$

Now as we have a one tail test

$$\text{p-value} = P(W_{MEN} \leq 13788.5) \approx 0.0003 < 0.05$$

Hence we are in the critical region, and we have reason to reject the null hypothesis.
We conclude that women are more concerned about food safety at Restaurants, than men.

Q5. Ant and Dec

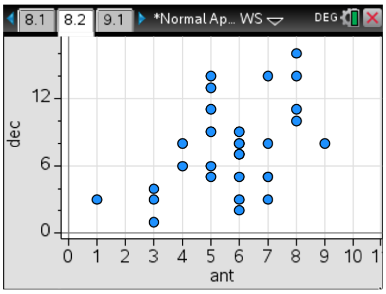
Thirty people rated each of the TV Presenters Anthony McPartlin and Declan Donnelly for their attractiveness.

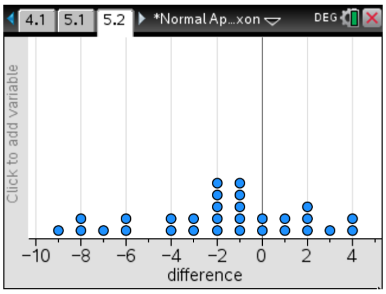
Ant	3	4	5	6	5	6	7	7	8	6	8	9	5	6	7
Dec	3	8	9	7	14	7	3	14	11	3	10	8	6	9	8
Ant	6	7	4	3	6	5	8	7	6	3	1	5	6	5	8
Dec	5	5	6	4	8	5	14	5	8	1	3	13	2	11	16

Establish who appears to be rated most highly and test if their lead is significant.

Summary Statistic:
W=100.5

Full Worked Solution to “Ant and Dec”





Data is paired.
If we assume that the distributions of the differences in the attractiveness scores are symmetrical (supported by the dot plot) then we can do a Wilcoxon Signed Rank Sum test.

We are asked to establish who is considered most attractive, and whether their lead is significant. Hence the dot plots suggest that Dec receives higher scores, so we shall perform a one-tail test that Dec scores more highly than Ant.

H_0: population $\text{median}_{\text{DIFFERENCE}}=0$ where difference = Ant score – Dec score
H_1: population $\text{median}_{\text{DIFFERENCE}}<0$ ie Dec scores more highly than Ant
$\alpha=5\%$. One tailed test.
Assume H_0 is true.

Ranking the data gives:

Ant	3	4	5	6	5	6	7	7	8	6	8	9	5	6	7	6	7	4	3	6	5	8	7	6	3	1	5	6	5	8
Dec	3	8	9	7	14	7	3	14	11	3	10	8	6	9	8	5	5	6	4	8	5	14	5	8	1	3	13	2	11	16
A-D	0	-4	-4	-1	-9	-1	4	-7	-3	3	-2	1	-1	-3	-1	1	2	-2	-1	-2	0	-6	2	-2	2	-2	-8	4	-6	-8
|A-D|	0	4	4	1	9	1	4	7	3	3	2	1	1	3	1	1	2	2	1	2	0	6	2	2	2	2	8	4	6	8
rank		20.5	20.5	4	28	4	20.5	25	17	17	11.5	4	4	17	4	4	11.5	11.5	4	11.5		23.5	11.5	11.5	11.5	11.5	26.5	20.5	23.5	26.5
Wpos							20.5			17		4				4	11.5						11.5		11.5			20.5		

We rejected the 2 tied results that gave an absolute difference of 0, leaving us with 28 pairs.
So, $W_{POS}=100.5$ and $Wneg=\frac{1}{2}\times 28\times 29-100.5=406-100.5=305.5$
We focus on the minimum, $W_{pos}=100.5$

We would reject H_0 for small values of W_{pos} or large values of W_{neg}

As n>20, we approximate W_{POS} with W=normal approximation to W_{POS},

$$W\text{~}N\left(\tfrac{1}{4}28\left(28+1\right),\tfrac{1}{24}28\left(28+1\right)\left(2\times 28+1\right)\right)$$
$$W\text{~}N\left(203,\tfrac{3857}{2}\right)$$

So, we want to know $P(W_{POS}\leq 100.5)$, as we reject H_0 for small values of W_{POS}

$$\begin{aligned}P(W_{POS}\leqslant 100.5) &\approx P(W<101)\ \textit{by continuity correction}\\ &=P\left(Z<\frac{101-203}{\sqrt{3857/2}}\right)\\ &=P(Z<-2.32269...)\\ &=0.010098..\ \textit{from normcdf}(-9E99,-2.32269)\\ &\approx 0.0101\,(4dp)\end{aligned}$$

We are in the 5% tail, and thus we can reject the null hypothesis and conclude that Dec does have a significant lead over Ant in the population median attractiveness scores.
Whey-Aye Man.